Grade 2 Mathematics Standards

Comparison Tool for Standards Transition

Updated March 2012

This document can be used to assist educators in analyzing the commonalities and differences between the new Alaska Mathematics standards and the Fourth Edition (Grade Level Expectations). This document is a first start toward a transition and districts may choose to augment with more detail.

The first column contains the new math standards. The second column shows the Grade Level Expectations (GLEs) that align to the new standards. The third column provides comments, usually highlighting differences between the new standards and GLEs that align in higher grades. Additionally, the comments may include a notation about an increase in rigor. Rigor may be defined as a standard that requires deeper understanding, higher order thinking, expanded analytical processes, or simply a skill introduced at an earlier grade.

Note that some GLEs are coded with an L. This signifies that the GLE was not assessed on the statewide assessment; it was to be assessed at the local level. No new standards are identified as being for local assessment. Students advancing through the grades are expected to meet each year’s grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

In most cases there are not complete matches between the two sets of standards, and it should not be assumed that either the content or skills found in one set of standards will match completely with those of the other set.

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| 6.RP.2. Understand the concept of a unit rate (*a*/*b* associated with a ratio *a:b* with *b ≠*0, and use rate language in the context of a ratio relationship) and apply it to solve real world problems (e.g., unit pricing, constant speed).  *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”* | **[6] E&C-5** developing and interpreting scale models  Any aligned GLE found in the higher grades will need to be absorbed in the lower grade as part of the transition. | Grade 6 GLE provides a specific real world model for understanding unit rate.  **[7] E&C-6** solving proportions using a given scale  **[8] E&C-5** using ratio and proportion |

The new standards represent a shift in the purpose of the standards. They are more instructional in nature, intended to guide classroom curriculum. The new standards do not serve as an assessment document unlike the GLEs. The Department with the support of stakeholders will prepare an assessment framework which will guide the development of the new assessments. The new standards will be assessed starting spring 2016. Until then, all districts will continue administering the Standards Based Assessments aligned to the GLEs through spring 2015.

A table at the end shows the GLEs not matched to the new standards. The comment column indicates where the GLE may be matched to a new standard in a lower or higher grade. Although some GLEs will be taught at other grade levels, teachers must provide opportunities for these GLEs to be reviewed in preparation for the spring Standards Based Assessments through spring 2015.

| **Grade 6 Math GLEs not matched by new standards** | **Comments** |
| --- | --- |
| **The student demonstrates conceptual understanding of fractions (proper or mixed numbers), decimals, percents (whole number), or integers by**  **[6] N-2** identifying place value positions from thousandths to millions (L) (M1.2.2) | 4th and 5th Grade Standards  **(4.NF.6, 4.NF.7, 5.NBT.3)** |

This GLE must be reviewed prior to the SBA through spring 2015.

Finally, the new standards for each grade level define what students should understand and be able to do by the end of each grade. They correspond to Standards of Mathematical Practice. The Standards of Mathematical Practice describe characteristics and traits that mathematics educators at all levels should seek to develop in their students. They describe ways that students should be engaging with mathematics as they progress through school. The integration of these standards into classroom instruction is a key strategy for increasing cognitive demand and conceptual learning. The Standards for mathematical Practice are included at the end of the document.

**Grade 2 Overview**

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| --- | --- |
| **Operations and Algebraic Thinking**  • Represent and solve problems involving  addition and subtraction.  • Add and subtract within 20.  • Work with equal groups of objects to gain  foundations for multiplication.  **Number and Operations in Base Ten**  • Understand place value.  • Use place value understanding and  properties of operations to add and subtract.  **Measurement and Data**  • Measure and estimate lengths in standard  units.  • Relate addition and subtraction to length.  • Work with time and money.  • Represent and interpret data.  **Geometry**  • Reason with shapes and their attributes. | **In Grade 2, instructional time should focus on four critical areas:**  (1) extending understanding of base-ten notation;  (2) building fluency with addition and subtraction;  (3) using standard units of measure; and  (4) describing and analyzing shapes. |
| **Mathematical Practices (MP)**   1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning. |

**Alaska New Mathematics Standards – Operations and Algebraic Thinking**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| **Operations and Algebraic Thinking 2.OA** |  |  |
| **Represent and solve problems involving addition and subtraction.** |  |  |
| 2.OA.1. Use addition and subtraction strategies to estimate, then solve one- and two-step word problems (using numbers up to 100) involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem. | **[2] E&C-2** estimating the results of simple addition and subtraction problems up to 100  **[2] E&C-5** solving two-digit addition and subtraction problems using a variety of models and algorithms  **[2] F&R-3** solving a problem with an unknown (e.g., 7 + ? = 10)  **[2] PS-3** translating problems from everyday language into math language and symbols  (+, -, =, <, >)  **[2] PS-5** using manipulatives, models, pictures, and language to represent and communicate mathematical ideas (M8.1.2) | The new standard specifies one- and two-step word problems with unknowns in all positions. |
| **Add and subtract using numbers up to 20.** |  |  |
| 2.OA.2. Fluently add and subtract using numbers up to 20 using mental strategies. Know from memory all sums of two one-digit numbers. | **[2] E&C-4** recalling addition and subtraction facts to 20 | The new standard requires “know from memory all sums of two one-digit numbers” and to fluently add and subtract. New standard 1.OA.6 list suggested mental strategies. |
| **Work with equal groups of objects to gain foundations for multiplication.** |  |  |
| 2.OA.3. Determine whether a group of objects (up to 20) is odd or even (e.g., by pairing objects and comparing, counting by 2s). Model an even number as two equal groups of objects and then write an equation as a sum of two equal addends. | **[2] N-7** identifying or using patterns in the number system (skip count by 2’s, 5’s, or 10’s; add or subtract by 10; identify even or odd numbers) | The new standard requires writing an equation of an even number. |
| 2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Write an equation to express the total as repeated addition (e.g., array of 4 by 5 would be 5 + 5 + 5 + 5 = 20). | NEW – not addressed in the GLEs | The GLEs do not use rectangular arrays. |
| **Identify and continue patterns.** |  |  |
| 2.OA.5. Identify, continue and label number patterns (e.g., aabb, abab). Describe a rule that determines and continues a sequence or pattern. | **[2] F&R-1** identifying and continuing patterns, including numbers  **[2] F&R-2** describing a rule or relation that determines and continues a sequence or pattern |  |

**Alaska New Mathematics Standards – Number and Operations in Base Ten**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
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| **Number and Operations in Base Ten 2.NBT** |  | GLEs do not have the equivalent focus of the Number and Operations in Base Ten domain. |
| **Understand place value.** |  |  |
| 2.NBT.1. Model and identify place value positions of three digit numbers. Include:  a. 100 can be thought of as a bundle of ten tens --called a "hundred".  b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). | **[2] N-2** modeling and identifying place value positions: ones, tens, and hundreds | The new standard states using bundles of ten tens as a “hundred.”  **[3] N-2** modeling (base ten blocks) or identifying place value positions to thousands |
| 2.NBT.2. Count up to 1000, skip-count by 5s, 10s and 100s. | **[2] N-7** identifying or using patterns in the number system (skip count by 2’s, 5’s, or 10’s; add or subtract by 10; identify even or odd numbers) | The new standard includes skip counting by 100. |
| 2.NBT.3. Read, write, order up to 1000 using base-ten numerals, number names and expanded form. | **The student demonstrates conceptual understanding**  • **of whole numbers to one thousand by**  **[2] N-1** reading, writing, ordering/counting and modeling correspondence of whole numbers  **[2] N-2** modeling and identifying place value positions: ones, tens, and hundreds | GLEs do not specify expanded form until grade 4.  **[4] N-3** converting between whole numbers expressed in expanded notation and standard form |
| 2.NBT.4. Compare two three-digit numbers based on the meanings of the hundreds, tens and ones digits, using >, =, < symbols to record the results. | **[2] F&R-4** using the terms equal to, greater than, and less than for numbers up to 100  **[2] PS-3** translating problems from everyday language into math language and symbols  (+, -, =, <, >) | The new standard asks students to compare three-digit numbers while the GLE only asks students to compare numbers up to 100 in grade 2. |
| **Use place value understanding and properties of operations to add and subtract.** |  |  |
| 2.NBT.5. Fluently add and subtract using numbers up to 100.  Use:   * strategies based on place value * properties of operations * and/or the relationship between addition and subtraction. | **[2] E&C-5** solving two-digit addition and subtraction problems using a variety of models and algorithms | The new standard includes “fluently.” |
| 2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations. | **[2] E&C-5** solving two-digit addition and subtraction problems using a variety of models and algorithms | The new standard specifies addition of up to four numbers. |
| 2.NBT.7. Add and subtract using numbers up to 1000.  Use:   * concrete models or drawings and strategies based on place value * properties of operations * and/or relationship between addition and subtraction.   Relate the strategy to a written method and explain the reasoning used.  Demonstrate in adding or subtracting three-digit numbers, hundreds and hundreds are added or subtracted, tens and tens are added or subtracted, ones and ones are added or subtracted and sometimes it is necessary to compose a ten from ten ones or a hundred from ten tens. | NEW – not addressed in the GLEs | Three-digit computation is addressed in grade 4.  **[4] E&C-3** adding or subtracting three-digit whole numbers  GLEs do not address composing numbers. |
| 2.NBT.8. Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number. | **[2] N-7** identifying or using patterns in the number system (skip count by 2’s, 5’s, or 10’s; add or subtract by 10; identify even or odd numbers) | GLEs do not address adding and subtracting 100 or three-digit numbers until grade 4, as shown above. |
| 2.NBT.9. Explain or illustrate the processes of addition or subtraction and their relationship using place value and the properties of operations. | **[2] N-5** describing or illustrating the processes of addition and subtraction of whole numbers and their relationships |  |

**Alaska New Mathematics Standards – Measurement and Data**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
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| **Measurement and Data 2.MD** |  |  |
| **Measure and estimate lengths in standard units.** |  |  |
| 2.MD.1. Measure the length of an object by selecting and using standard tools such as rulers, yardsticks, meter sticks, and measuring tapes. | **[2] MEA-5** selecting and using appropriate tools of measurement |  |
| 2.MD.2. Measure the length of an object twice using different length units for the two measurements. Describe how the two measurements relate to the size of the unit chosen. | NEW – not addressed in the GLEs |  |
| 2.MD.3. Estimate, measure and draw lengths using whole units of inches, feet, yards, centimeters and meters. | **[2] MEA-1** measuring to the nearest inch or foot | The GLE does not ask students to estimate or include yards, centimeters, and meters.  **[3] MEA-1** estimating length to the nearest inch or foot (L)  **[3] MEA-4** selecting an appropriate unit of English, metric, or non-standard measurement to estimate the length, time, weight, or temperature  **[4] MEA-1** [estimating length to the nearest half-inch or centimeter L]  **[4] MEA-3** identifying or using equivalent measures for length (inch, foot, yard: 12 inches = 1 foot, 3 feet = 1 yard, 36 inches = 1 yard; centimeter, meter: 100 centimeters = 1 meter) |
| 2.MD.4. Measure to compare lengths of two objects, expressing the difference in terms of a standard length unit. | **[2] MEA-1** measuring to the nearest inch or foot  **[2] MEA-2** comparing and ordering objects by length, weight, area, time, temperature | The new standard asks students to express the difference in lengths. |
| **Relate addition and subtraction to length.** |  |  |
| 2.MD.5. Solve addition and subtraction word problems using numbers up to 100 involving length that are given in the same units (e.g., by using drawings of rulers). Write an equation with a symbol for the unknown to represent the problem. | NEW – not addressed in the GLEs |  |
| **Work with time and money.** |  |  |
| 2.MD.6. Tell and write time to the nearest five minutes using a.m. and p.m. from analog and digital clocks. | **[2] MEA-7** telling time to the nearest **¼** hour using analog and digital clocks | New standard specifies to the nearest 5 minutes and using a.m. and p.m., which are addressed in grade 4 and 5 GLEs. **[4] MEA-6** telling time in 5 minute increments using analog clocks**[5] MEA-5** telling time using analog clocks to the nearest minute and using AM or PM |
| 2.MD.7. Solve word problems involving dollar bills and coins using the $ and ¢ symbols appropriately. | **[2] MEA-4** identifying coins, their value, or the value of a set of coins up to one dollar  **[2] MEA-10** counting change (coins) up to a dollar  **[2] MEA-11** recognizing money symbols including a decimal point ($,¢, .)  **[2] MEA-12** identifying equal values of coins up to a dollar | The new standard specifies solving word problems involving money. |
| **Represent and interpret data.** |  |  |
| 2.MD. 8. Collect, record, interpret, represent, and describe data in a table, graph or plot. | **[2] S&P-2** collecting, recording, interpreting, and representing data in a variety of ways  **[2] S&P-3** describing data from a variety of graphs (e.g., newspapers, magazines, texts, computers, and other sources) |  |
| 2.MD.9. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart and compare problems using information presented in a bar graph. | **[2] S&P-1** constructing a variety of graphs from realistic situations  **[2] S&P-2** collecting, recording, interpreting, and representing data in a variety of ways  **[2] S&P-3** describing data from a variety of graphs (e.g., newspapers, magazines, texts, computers, and other sources) | The new standard specifies three types of problems: simple put together, take-apart and compare. |

**Alaska New Mathematics Standards – Geometry**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
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| **Geometry 2.G** |  |  |
| **Reason with shapes and their attributes.** |  |  |
| 2.G.1. Identify and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces compared visually, not by measuring. Identify triangles, quadrilaterals, pentagons, hexagons and cubes. | **[2] G-1** describing attributes of a triangle, circle, square, and rectangle  **[2] G-11** drawing, copying, or describing a variety of shapes | New standard specifies more shapes, including quadrilaterals, pentagons, hexagons and cubes. These are in grade 3 and grade 5 GLEs.  **[3] G-2** using the attributes and properties of plane figures to [model L], identify, compare, or describe plane figures (circles, rectangles, squares, and triangles)[and solid figures (cubes, cylinders, or spheres) L]  **[5] G-1** using the attributes and properties of angles and the number, length, and orientation of sides to identify or compare triangles (scalene, isosceles, or equilateral) or quadrilaterals (parallelograms, trapezoids, rhombi) |
| 2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. | **[2] G-8** determining perimeter and area of rectangular shapes using grid paper and/or manipulatives |  |
| 2.G.3. Partition circles and rectangles into parts, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. | **[2] N-3** identifying fractions as equal parts of a whole, a region, or a set | The new standard includes “recognize that equal shares of identical wholes need not have the same shape.” |

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| **Grade 2 Math GLEs Not Matched by New Standards** | **Comment** | |
| **The student demonstrates conceptual understanding of simple fractions**  **[2] N-4** reading and writing numerals for simple fractions | Fractions addressed in new 3rd Grade standards. | |
| **The student demonstrates conceptual understanding of number theory by**  **[2] N-6** modeling or explaining the commutative and identity properties of addition | New 1st Grade standard (**1.OA.3**) | |
| **[2] N-8** modeling fact families | New 1st Grade standard **(1.OA.6)** | |
| **The student demonstrates understanding of measurable attributes by**  **[2] MEA-3** comparing objects to standard and nonstandard units to identify objects that are greater than, less than, and equal to a given unit | New 1st Grade standard **(1.MD.1)** | |
| **The student demonstrates ability to use measurement techniques by**  **[2] MEA-6** drawing a line segment to the nearest half inch | New 3rd Grade standard **(3.MD.5)** which includes measuring to half an inch is related. | |
| **[2] MEA-8** ordering the months of the year | New 1st Grade standard **(1.MD.4)** involves reading a calendar is related. | |
| **[2] MEA-9** writing the date using words and numbers (day, month, year) | New 1st Grade standard **(1.MD.4**). | |
| **The student determines reasonable answers to real-life situations, paper/pencil computations, or calculator results by**  **[2] E&C-1** estimating “how many” and “how much” in a given set up to 30 | New 1st Grade standard **(1.CC.6**) limits the estimation up to 20. | |
| **[2] E&C-3** identifying whether estimation or counting is appropriate | Not specifically addressed in new standards. | |
| **The student accurately solves problems (including real-world situations) involving**  **[2] E&C-6** using repeated addition with objects to model multiplication | New 2nd Grade standard **(2.OA.4**) including using repeated addition is related. | |
| **[2] E&C-7** using equal shares with objects to model division | Division is addressed in new 3rd Grade standards. | |
| **The student demonstrates an understanding of geometric relationships by**  **[2] G-2** identifying and classifying 3-dimensional shapes (e.g., cone, sphere and cylinder) | New Kindergarten standards **(K.G.2, K.G.3)**. | |
| **[2] G-3** relating real-world examples to the ideas and concepts of geometry | Not specifically addressed in new standards. | |
| **[2] G-4** constructing, comparing, classifying, and describing the relationship among geometric figures | New 1st Grade standard **(3.G.1)**. Construction addressed fully in new 7th Grade standards. | |
| **The student demonstrates conceptual understanding of similarity, congruence, symmetry, or transformations of shapes by**  **[2] G-5** creating simple shapes using concrete materials/manipulatives | Similarity, congruence and transformations fully addressed in new 8th Grade standards. | |
| **[2] G-6** identifying or drawing lines of symmetry for simple shapes | New 4th Grade standard **(4.G.3**). | |
| **The student solves problems using perimeter or area by**  **[2] G-7** explaining the difference between perimeter and area | New 3rd Grade standard **(3.MD.10)**. | |
| **The student demonstrates understanding of position and direction by**  **[2] G-9** describing relative locations of objects using directional terms (inside, outside, right, left) | New Kindergarten standard **(K.G.1**) is related but limited to relative positions not directional. | |
| **[2] G-10** creating a simple map to show location of objects | Not specifically addressed in the new standards. | |
| **The student demonstrates a conceptual understanding of probability and counting techniques by**  **[2] S&P-4** predicting, interpreting, and comparing data using events or repeated observations) | Probability addressed in new 6th Grade standards. | |
| **[2] S&P-5** recognizing the difference between chance and certainty |
| **The student demonstrates an ability to problem solve by**  **[2] PS-1** creating and solving a variety of problems using appropriate strategies | The GLE math process skills are incorporated in to the Standards for Mathematical Practice.   1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.     Descriptions of the Standards for Mathematical Practice follow this chart as well as the grade-span descriptors appropriate to this grade level. |
| **The student demonstrates an ability to problem solve by**  **[2] PS-2** choosing appropriate operations to solve a given problem |
| **The student communicates his or her mathematical thinking by**  **[2] PS-4** using everyday language to explain thinking about problem solving strategies and solutions to problems |
| **The student demonstrates an ability to use logic and reason by**  **[2] PS-6** explaining why a prediction, estimation, or solution is reasonable |
| **The student demonstrates an ability to use logic and reason by**  **[2] PS-7** drawing pictures that support or refute mathematical statements |
| **The student understands and applies mathematical skills and processes across the content strands by**  **[2] PS-8** using real world context (e.g., self, friends, and family) |

**Alaska New Standards for Mathematical Practice**

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| **1. Make sense of problems and persevere in solving them.**  Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. | **In grades K-2 mathematically proficient students will:**   1. focus on the problem and check for alternate methods 2. check if the solution makes sense |
| **2. Reason abstractly and quantitatively.**  Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. | **In grades K-2 mathematically proficient students will:**   1. represent a situation symbolically and/or with manipulatives 2. create a coherent representation of the problem 3. use units of measurement consistently |
| **3. Construct viable arguments and critique the reasoning of others.**  Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. | **In grades K-2 mathematically proficient students will:**   * construct arguments using concrete referents such as objects, drawings, diagrams, and actions * justify conclusions, communicate conclusions * listen to arguments and decide whether the arguments make sense |
| **4. Model with mathematics.**  Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two‐way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. | **In grades K-2 mathematically proficient students will:**   * apply mathematics to solve problems in everyday life * identify important quantities in a practical situation and model the situation with manipulatives or pictures * interpret mathematical results in the context of the situation and reflect on whether the results make sense |
| **5. Use appropriate tools strategically.**  Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. | **In grades K-2 mathematically proficient students will:**   * select the available tools (such as pencil and paper, manipulatives, rulers, and available technology) when solving a mathematical problem * be familiar with tools appropriate for the grade level to make sound decisions about when each of these tools might be helpful * identify relevant external mathematical resources and use them to pose or solve problems * use technological tools to explore and deepen their understanding of concepts |
| **6. Attend to precision.**  Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. | **In grades K-2 mathematically proficient students will:**   * give thoughtful explanations to each other * use clear definitions and reasoning in discussion with others * state the meaning of symbols they choose, including using the equal sign consistently and appropriately |
| **7. Look for and make use of structure.**  Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3*(x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. | **In all grade levels mathematically proficient students will:**   * discern a pattern or structure * understand complex structures as single objects or as being composed of several objects * check if the answer is reasonable |
| **8. Look for and express regularity in repeated reasoning.**    Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (*y* – 2)/(*x* – 1) = 3. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. | **In all grade levels mathematically proficient students will:**   * identify if calculations or processes are repeated * use alternative and traditional methods to solve problems * evaluate the reasonableness of their intermediate results, while attending to the details |